Numerical simulation of wave propagation in Y- and Z-type hexaferrites for high frequency applications

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The dispersion characteristics of microstrip lines on magnetically anisotropic substrates are studied utilizing the Galerkin’s method in the spectral domain. Specifically, we calculate the differential phase shifts of devices consisting of Z-type (Zn$_2$Z) and Y-type (Zn$_2$Y) ferrites as a function of applied magnetic field. Our theoretical formulism is effective in modeling highly magnetically anisotropic materials, which are becoming increasingly important for the next generation of microwave devices. Such a theoretical treatment of anisotropic ferrite magnetic materials is presently unavailable in commercial numerical simulation tools. © 2010 American Institute of Physics. [doi:10.1063/1.3338970]

I. INTRODUCTION

Phase shifters are utilized in a variety of applications, including phased array radar and many wireless communication systems. While several different technologies have emerged over the years, such as semiconductor, ferroelectric, and microelectromechanical systems devices, ferrite phase shifters are still recognized for their exceptional insertion loss and high power handling capabilities.\textsuperscript{1–6} For operation at microwave frequencies, low loss ferrites such as yttrium garnets and lithium spinels require strong magnetic bias fields that can be realized with a combination of permanent magnets and current driven coils. The magnitude of the magnetic bias field necessary to operate a ferrite device at high frequency can be greatly reduced if an anisotropic material, such as hexagonal Z- or Y-type ferrites, is utilized. The strong magnetocristalline anisotropy field in these materials can be used to compensate for the external magnetic bias field required.\textsuperscript{7} Cubic ferrites, such as yttrium iron garnet (YIG), require 3000–4000 Oe to operate at the same frequency band. As such, anisotropic magnetic materials are expected to play an important role in the design and development of future microwave systems as they allow greatly reduced component size, weight, cost, and dc power consumption.

Unfortunately, commercial numerical simulation packages, such as Computer Simulation Technology MICROWAVE STUDIO\textsuperscript{8} and Ansoft HFSS\textsuperscript{9}, are yet to allow magnetically anisotropic materials to be included in microwave device modeling. Therefore, there is a strong need for numerical methods to accurately simulate the performance of structures that include hexagonal Z- and Y-type ferrites. One such method is presented in this paper.

The permeability in magnetic materials assumes a tensor form. For ferrites that posses a cubic crystal structure, such as garnets or spinels, the precessional motion of the magnetization vector under an applied magnetic bias field is circular, with the diagonal components of the permeability tensor being equal. This is not the case in hexagonal ferrites where the motion is elliptical due to strong magnetocristalline anisotropy fields. As such, the diagonal elements of the permeability tensor are no longer equal. The easy plane is the x-z plane, normal to the c axis, which is along the y axis. The microwave permeability tensor of a hexagonal Z- or Y-type ferrite magnetized in the direction perpendicular to the crystallographic c axis is given in the CGS system of units by

\[
\begin{bmatrix}
\mu_{xx} & \mu_{xy} & 0 \\
\mu_{yx} & \mu_{yy} & 0 \\
0 & 0 & \mu_{zz}
\end{bmatrix},
\]

where

\[
\mu_{xx} = 1 + \frac{4\pi M_s (H + H_A)}{H(H + H_A) - \frac{\omega^2}{\gamma}},
\]

\[
\mu_{yy} = 1 + \frac{4\pi M_H}{H(H + H_A) - \frac{\omega^2}{\gamma}},
\]

\[
\mu_{xy} = \frac{4\pi M_s \omega}{H(H + H_A) - \frac{\omega^2}{\gamma}},
\]

\[
\mu_{zz} = 1,
\]

where $4\pi M_s$ is the saturation magnetization, $M_A$ is the magnetocristalline anisotropy field, $H$ is the internal field, $\omega$ is the radial frequency, and $\gamma$ is the electron gyromagnetic ratio. Equation (1) is applicable for a semi-infinite medium of Z- or Y-type hexaferrite.

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Galerkin’s method has been successfully applied in the analysis of microstrip lines on scalar permittivity substrates by Itoh.\textsuperscript{8} This method was later extended to the analysis of microstrip and slotlines on anisotropic permittivity substrates by Geshiro.\textsuperscript{9} At the same time, isotropic ferrite materials was analyzed by Kitazawa and Itoh using the spectral domain approach.\textsuperscript{10} In this paper, Galerkin’s method is applied to microstrip lines on anisotropic hexagonal Z- and Y-type ferrite substrates. Resulting dispersion characteristics are used in the analysis of a phase shifter device, which allows for the evaluation of key design parameters, such as phase constant and differential phase shift. The structure of the ferrite phase shifter is shown in Fig. 1.

\section*{II. MICROSTRIP LINE STRUCTURE AND FORMULATION}

The structure under consideration is a microstrip line on top of a longitudinally biased Z- or Y-type ferrite substrate with the crystallographic $c$ axis aligned perpendicular to the plane and a ground plane on the bottom of the substrate. We define a Fourier transform between the spatial coordinate $x$ and the spectral domain parameter $\alpha$,

\begin{equation}
\tilde{f}(\alpha, y) = \int_{-\infty}^{+\infty} f(x, y) e^{i\alpha x} dx,
\end{equation}

where $f(x, y)$ represents any component of either the electric or the magnetic field and $e^{-i\beta z}$ dependence in the propagation direction is assumed.

After simplification, the wave equations for the longitudinal field components $\tilde{E}_z(\alpha, y)$ and $\tilde{H}_z(\alpha, y)$ are derived from Maxwell’s equations as

\begin{equation}
\begin{aligned}
\alpha^2 \tilde{E}_z + (\mu_y - \mu_{yy}) \frac{\partial^2}{\partial y^2} \tilde{E}_z + \mu_{zz} \frac{\partial^2}{\partial z^2} \tilde{E}_z + \mu_{yy} \frac{\partial^2}{\partial x^2} \tilde{E}_z &= 0, \\
\mu_{yy} \frac{\partial^2}{\partial y^2} \tilde{H}_z - \mu_{zz} \frac{\partial^2}{\partial z^2} \tilde{H}_z - \mu_{xx} \frac{\partial^2}{\partial x^2} \tilde{H}_z &= 0
\end{aligned}
\end{equation}

where

\begin{equation}
\begin{aligned}
&\mu_{xx} = e_{\|}\mu_{xx} - \beta^2, \\
&\mu_{yy} = e_{\perp}\mu_{yy} - \beta^2, \\
&\mu_{zz} = e_{\perp}\mu_{zz} - \beta^2, \\
&\mu_z = \mu_{xx}^2 + \mu_{yy}^2 + \mu_{zz}^2.
\end{aligned}
\end{equation}

Assume that $\tilde{E}_z(\alpha, y)$ and $\tilde{H}_z(\alpha, y)$ have an $e^{\gamma y}$ dependence in the transverse direction, where $\gamma$ can be solved from Eq. (5).

\begin{equation}
\begin{aligned}
(\mu_{xx}'\alpha^2 - \mu_{yy}'\mu_{zz}' - \beta^2)\frac{\partial^2}{\partial y^2} \tilde{E}_z + (\mu_{yy}'\mu_{zz}' - \mu_{xx}') \frac{\partial^2}{\partial x^2} \tilde{E}_z + 2\mu_z \frac{\partial^2}{\partial x^2} \tilde{E}_z - (\mu_{xx}'\mu_{yy}') \gamma^2 + (\mu_{yy}'\mu_{zz}' + e_{\|}\mu_{xx}'\beta^2) \gamma^2 &= 0
\end{aligned}
\end{equation}

Other field components can be obtained from $\tilde{E}_z(\alpha, y)$ and $\tilde{H}_z(\alpha, y)$. The electric and magnetic field components in air region can be obtained by setting $\mu_{xx} = \mu_{yy} = 1$, $\mu_{zz} = 0$, and $e_r = 1$. After applying the boundary conditions on the interface between ferrite and ground ($y=0$) and interface between ferrite and air ($y=d$), we can obtain

\begin{equation}
\begin{pmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22}
\end{pmatrix}
\begin{pmatrix}
\tilde{J}_e \\
\tilde{J}_z
\end{pmatrix}
= \begin{pmatrix}
\tilde{E}_e \\
\tilde{E}_z
\end{pmatrix}.
\end{equation}

Finally, Galerkin’s method is used to obtain a linear equation group, from which, by setting the determinant to be equal zero, the phase constant $\beta$ is derived.

\section*{III. RESULTS AND DISCUSSION}

Our proposed method was simplified ($H_z=0$) so that we could verify our results against previously published data. A good agreement was observed between dispersion characteristics obtained by our method and those obtained by the finite element method\textsuperscript{11} and the infinite line method\textsuperscript{12} for microstrip lines on magnetically isotropic YIG substrates. For example, a phase constant of 452.2 rad/m was calculated for applied field of 300 Oe by the proposed method, compared to 445 rad/m in Ref. 11 and 452.2 rad/m in Ref. 12. In the following, the numerical results for a microstrip phase shifter on anisotropic Z- and Y-type ferrite substrates are presented and compared with each other. The parameters of the device used in numerical calculations were $d=0.6$ mm, $w=0.3$ mm, and $4\pi M_s=2000$ Oe for Y-type ferrite, $4\pi M_s=3000$ Oe for Z-type ferrite, $H_A=9000$ Oe, ferromagnetic resonance linewidth $\Delta H=25$ Oe, and relative permittivity $e_r=19$.

Phase constant $\beta$’s of the Z- and Y-type ferrite substrate phase shifters for different values of the magnetic bias field are shown in Fig. 2. At high frequencies ($f>15$ GHz for Y-type ferrite and $f>18$ GHz for Z-type ferrite, ), the curves are nearly linear since the permeability of the anisotropic...
ferrites at these frequencies asymptotically approaches unity and the slope is determined predominantly by the dielectric constant. Phase constant dispersions exhibit cut-off behavior at low frequencies corresponding to the ferromagnetic antiresonance frequency of the ferrite. The frequency range between ferromagnetic resonance and antiresonance is characterized by negative permeability responsible for the cut-off behavior. Near the antiresonance frequency, the permeability changes rapidly, and by tuning the magnetic bias field, differential phase shifts can be realized. Figure 3 shows the differential phase shift calculated numerically. We define differential phase shifts of $S_{21}$ as follows:

$$\Delta \phi(H) = \phi_{21}(H) - \phi_{21}(H = 0),$$

where $H \leq 100$ Oe. As shown in this figure, Z-type ferrite exhibits higher phase shift and higher operating frequencies but lower bandwidth compared with Y-type ferrite under the same bias field. The explanation for the increased phase shift is due to the fact that Z-type ferrite has higher saturation magnetization than Y-type ferrite, resulting in steeper phase constant curve of Z-type ferrite above cutoff frequency. Figure 4 shows the tuning factor of the phase shifter. The tuning factor is defined as the amount of phase shift incurred per unit length per unit bias field. In Fig. 4, the tuning factor is higher at low frequency (near cut-off) and decreases monotonically with increasing frequency for both Z- and Y-type ferrites. Therefore, there is a trade-off between the tuning factor and bandwidth in practical devices.

**IV. CONCLUSIONS**

In this paper, the Galerkin’s method in spectral domain is utilized to determine the dispersion characteristics of microstrip lines on magnetically anisotropic substrates. The proposed approach is applied to analyze a magnetically tunable microstrip phase shifter on hexagonal Z- and Y-type ferrite substrates. The phase constant, differential phase shift, and tuning factor are calculated as a function of applied field. Future work will focus on the analysis of losses in devices, including magnetic, dielectric, and conductor losses.