Role of Ferrites in Negative Index Metamaterials

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Metamaterial composites consisting of copper wires and yttrium iron garnet (YIG) slabs have been theoretically studied using the transfer function matrix technique and predictions of electromagnetic scatterings have been compared well with experiments. An array of copper wires (without ferrite) showed plasma-like behavior in the calculation and was confirmed experimentally. The addition of a YIG slab demonstrated the effect of negative index. We believe that such a ferrite-wire metamaterial should provide low-loss and tunable negative index media (NIM) structures.

***Index Terms*—Ferrite, metamaterial, negative index media (NIM), tunable.**

I. INTRODUCTION

OVER the past 25–30 years, there has been great interest in periodic composite structures consisting of magneto-dielectric constituents, as well as metallic wires, flakes, etc. The motivation of these studies was to determine a particular configuration which may lead to electrical and/or magnetic properties which naturally cannot be obtained from single constituents. One configuration that appeared to be very promising consisted of periodic distribution of wires and rings [1], [2] in either one or two dimensions. These composite structures were referred to as left handed media. It was shown that the periodic wires structure led to effective negative permittivity [3]. Furthermore, split ring resonators (SRR) gave rise to negative permeability [4]. However, when these two structures were combined, such that the electromagnetic electric field E_{rf} was parallel to the wire direction and the electromagnetic magnetic field H_{rf} was perpendicular to the SRR elements or the ring, there resulted an effective negative index [5], n. Potential applications including imaging [6] and microwave technologies [7]–[9] have been explored. The purpose of this paper is to replace the SRR structure by introducing ferrites as a means to produce negative μ. The negative index may be obtained by having both wires/film and ferrites in a meta-material ferrite structure [10]–[12]. Such a structure is vastly more practical in shape and size than the wire-SRR construct.

II. CALCULATION METHOD

The metamaterial ferrite structure in transmission line consists of an array of copper wires and ferrite films. Copper wires are electrically connected to the transmission line, ferrite films are placed between the wires. We assume a transverse electromagnetic (TEM) mode of propagation such that E_{rf} is parallel to the wire axis and H_{rf} is perpendicular to the wire. The magnetization of the ferrite M is also along the wire direction. The diameter of the wire is sufficiently large compared to the skin depth, so that there is no wave propagation within the wire. We represent the wire element as a lumped element so that its transfer function matrix [13] is

$$A_w = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \tag{1}$$

where

$$Y = \left(\frac{l}{2\pi r} \sqrt{\frac{\omega \mu_0}{2\sigma}} (1+j) + j\omega\mu_0 l \frac{\ln(d/r)}{2\pi} \right)^{-1}$$

σ is conductivity, μ_0 is permeability of air, l is length of wire, r is the radius of the wire, d is the distance between the wires, and ω is the angular frequency. The admittance of the wire Y comes from two parts, surface impedance and self-inductance. Surface impedance is related to the electromagnetic field within the wire and self-inductance is related to the electromagnetic field around wire. The region between the wires is treated as a continuous ferrite medium and as such we write

$$A_f = \begin{bmatrix} \cos(kd) & jZ\sin(kd) \\ j\sin(kd)/Z & \cos(kd) \end{bmatrix} \tag{2}$$

where

$$Z = \sqrt{\frac{\mu}{\varepsilon}}, k = \omega\sqrt{\mu\varepsilon}, \varepsilon = \text{dielectric constant}$$

$$\mu = \frac{\mu_{xx}^2 + \mu_{xy}^2}{\mu_{xx}}$$

and $d =$ distance between wires. There is no cross term between the transfer function matrix of wire and ferrite medium, because we assume there is no direct contact between them. The transfer function matrix representing the N layers of metamaterial and ferrite structure is then

$$[A] = \{[A_w]\cdot[A_f]\}^N \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \tag{3}$$

From the determination of the a_{ij}'s the scattering S-parameters may be easily calculated and they are

$$S_{21} = \frac{2Z_0}{a_{12} + (a_{11}+a_{22})Z_0 + a_{21}Z_0^2} \tag{4}$$

$$S_{11} = \frac{a_{12} + (a_{11}-a_{22})Z_0 - a_{21}Z_0^2}{a_{12} + (a_{11}+a_{22})Z_0 + a_{21}Z_0^2} \tag{5}$$

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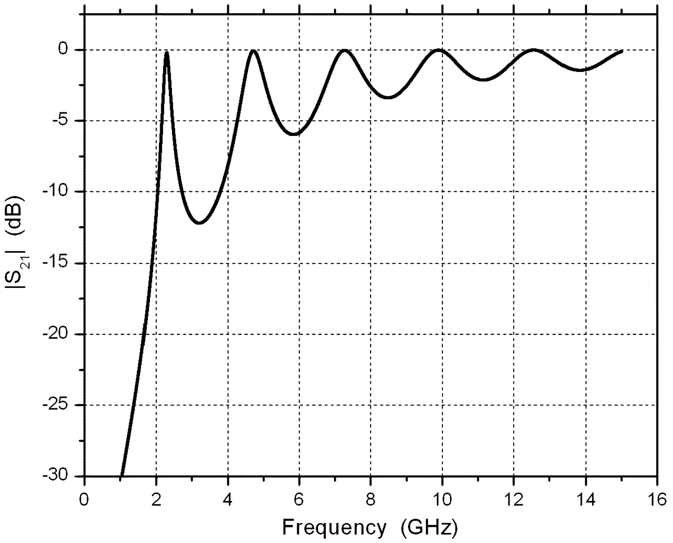

Fig. 1. Transmittance of two copper wires (5.5 cm apart) connected to the transmission line calculated by transfer matrix theory.

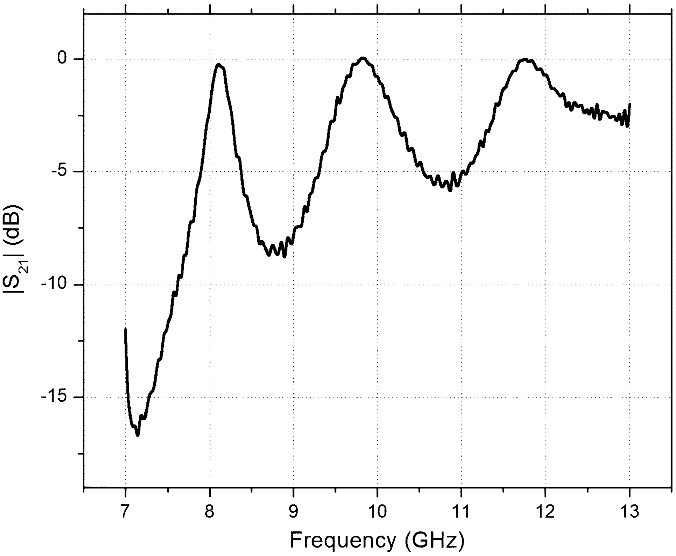

Fig. 2. Experimentally measured transmittance of two copper wires (5.5 cm apart) in an X-band waveguide.

where Z_o is the characteristic impedance of the medium at the input and output of the metamaterial ferrite composite. The effective refractive index n_{eff} and impedance Z_{eff} may now be calculated in terms of S_{11} and S_{21} and is given as [14]

$$n_{\mathrm{eff}} = \pm \frac{c}{2\pi \cdot f \cdot L} \cdot \cos^{-1}\left(\frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}}\right) \tag{6}$$

$$Z_{\mathrm{eff}} = \pm\sqrt{\frac{(1+S_{11})^2 - S_{21}^2}{(1-S_{11})^2 - S_{21}^2}}. \tag{7}$$

The effective permittivity and permeability can be obtained by $\varepsilon_{\mathrm{eff}} = n_{\mathrm{eff}}/Z_{\mathrm{eff}}, \mu_{\mathrm{eff}} = n_{\mathrm{eff}} \cdot Z_{\mathrm{eff}}$.

III. RESULTS

We tested our calculation for the simple case of having only two copper wires without a ferrite inserted. The spacing between the wires was 5.5 cm. Fig. 1 shows the theoretical calculation of the transmittance, $|S_{21}|$. It clearly shows the periodic structure in frequency domain due to the resonance between the two wires.

The frequency of the first maximum transmittance is the so-called plasma frequency ω_p. Below this frequency, the composite has an effective negative permittivity $\varepsilon_{\mathrm{eff}} \approx 1 - ((\omega_p/\omega))^2$. This was compared to the experimental result (Fig. 2) of two copper wires with the same spacing in an X-band waveguide. Although the theory shows the correct resonance phenomena, the oscillations dampen at high frequencies. This is because we assume the wire is a lumped element and this approximation is not valid at high frequencies.

It is well known that the ferromagnetic resonance (FMR) can be tuned by the application of an external magnetic field H. Accordingly, the range of frequencies by which μ is negative is given by

$$\Delta f = f_0 - f_r \approx \frac{\gamma}{2\pi}(2\pi M_s) \tag{8}$$

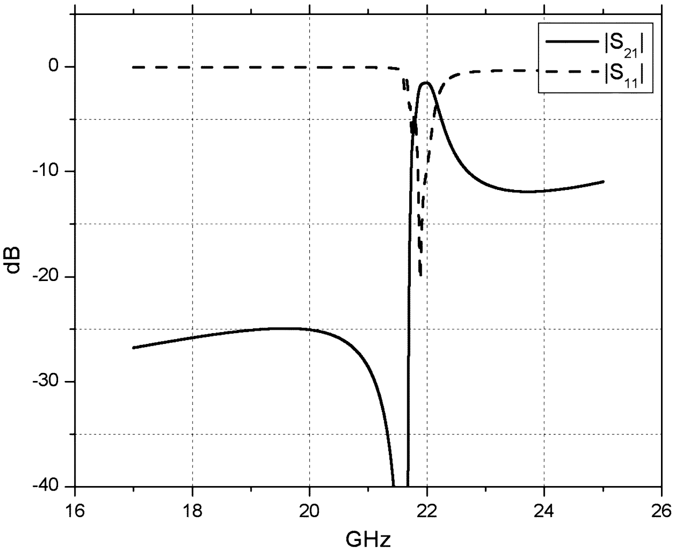

Fig. 3. Theoretical calculations of S parameters for the metamaterial wires and ferrite. Composite structure consisted of eight copper wires spaced 1 mm apart and YIG films with a bias field of 6.9 kOe. Solid line represents the magnitude of S_{21} while the dashed line represents the magnitude of S_{11}.

where $f_r = (\gamma/2\pi)\sqrt{H(H + 4\pi M_s)}, \gamma/2\pi = 2.8$ GHz/KOe, $4\pi \mathrm{M}_s = 1750$ G, $f_r =$ FMR frequency, and $f_0 \approx f_r + (\gamma/2\pi)(2\pi M_s)$. The subscript "$_0$" is to denote the anti-resonance frequency at which $\mu = 0$. The object of tuning is to vary H such that f_r is varied according to (8), and, therefore, the onset of the frequency range at which micrometer is negative can be shifted with H. However, the range of frequencies for which micrometer is negative is approximately constant and is given as $(\gamma/2\pi)(2\pi M_s)$. The factor of $2\pi M_s$ is a result of applying H in the film plane. If H was applied perpendicular to the film plane, Δf would scale as $4\pi \mathrm{M}_\mathrm{s}$.

Fig. 3 shows the theoretical calculation of S parameters for the metamaterial wires and ferrite composite. In this calculation, we have used the following parameters: number of wires $= 8, \sigma = 5.8 \times 10^7$ mhos/m, $l = 0.43$ cm, $r = 12.5 \mu$m, $d = 1$ mm,

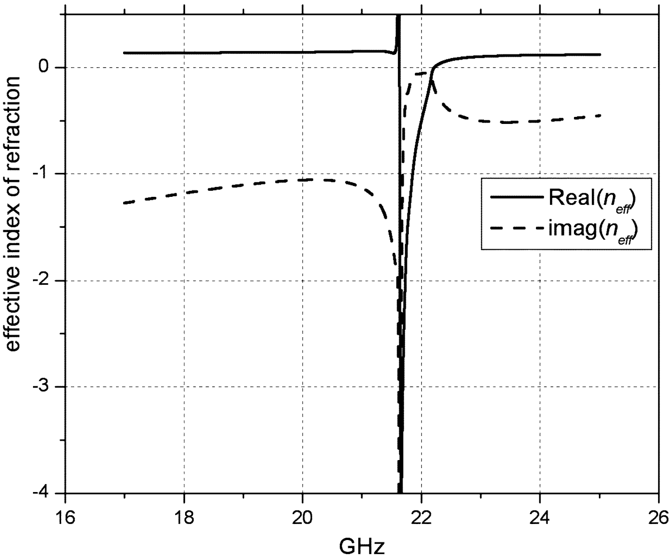

Fig. 4. Effective index of refraction retrieved from the S parameters in Fig. 3. Solid line represents the real part while the dashed line represents the imaginary part.

$4\pi M_s = 1750$ G, $\Delta\mathrm{H} = 10$ Oe, and $H = 6.9$ kOe. The minimal insertion loss determined in Fig. 3 is 2 dB/cm. The minimal insertion loss can be further reduced by using high values of $(4\pi M_s/\Delta H)$ and $(\omega L/R)$, where L is the inductance and R is the resistance of the single wire. Fig. 4 shows effective index of refraction retrieved from the S parameters in Fig. 3. Indeed, as is shown in Fig. 4, a negative index can be achieved with the potential of frequency tuning which has not be possible with the previously explored SRR constructs. Clearly, if we are to realize a negative index, we require both μ and ε to be negative. This means that irrespective of H, f_0 should not exceed the plasma frequency of the wires. If we choose $2\pi f_0 = \omega_p$, the maximum applied field used for tuning is: $H_{\max} \approx (\omega_p/\gamma) - 2\pi M_s$. Note that it may not been possible to tune ω_p, if only conductive wires are used. However, this does not pose a significant limitation to practical devices since the region of negative ε is broadband.

IV. CONCLUSION

The theoretical analysis method used in this paper, the transfer function matrix technique, is sufficient to explain the negative index phenomena. An array of copper wires (without ferrite) showed plasma-like behavior in the calculation and was confirmed by experimental results. With the addition of yttrium iron garnet (YIG) slab we demonstrated a negative index metamaterial employing ferrite materials and copper wires. Such a ferrite-wire metamaterial should provide a means to fabricate a low-loss, tunable NIM.

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